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that is, $x^2:(x^2+a^2-2cx)=(x+a)^2:d^2$.

$$\therefore d^2=\frac{(x+a)^2(x^2-2cx+a^2)}{x^2}=(x+a)^2\left(1-\frac{2c}{x}+\frac{a^2}{x^2}\right)$$

$$=x^2+2(a-c)x+2a(a-2c)+2a^2(a-c).\frac{1}{x}+\frac{a^4}{x^2}.$$

Also solved by G. B. M. Zerr.

CALCULUS.

286. Proposed by R. D. CARMICHAEL, Princeton University.

Solve the differential equation

$$\begin{aligned}&[a_0x^3+a_1x^2y+a_2xy^2+(a_0-a_1+a_2)y^3\\ &\qquad+a_3x^2+a_4xy+a_5y^2+a_6x+a_7y+a_8]dx\\ &+[a_0y^3+a_1xy^2+a_2x^2y+(a_0-a_1+a_2)x^3\\ &\qquad+a_3y^2+a_4xy+a_5x^2+a_6y+a_7x+a_8]dy=0.\end{aligned}$$

I. Solution by W. W. BEMAN, Prosessor of Mathematics, University of Michigan, Ann Arbor, Mich.

Putting $x=s+t$, $y=s-t$, and afterward $t^2=w$, the equation takes the linear form

$$\frac{dw}{ds}+\frac{4(3a_0-2a_1+a_2)s+2(a_3-a_4+a_5)}{4(a_1-a_2)s^2+2(a_3-a_5)s+a_6-a_7}.w$$

$$=-\frac{4(a_0+a_2)s^3+2(a_3+a_4+a_5)s^2+2(a_6+a_7)+2a_8}{4(a_1-a_2)s^2+2(a_3-a_5)s+a_6-a_7}.$$

Or, putting $x+y=2u$, $xy=2v$, the equation takes the linear form

$$\frac{dv}{du}+\frac{4(3a_0-2a_1+a_2)u+2(a_3-a_4+a_5)}{4(a_1-a_2)u^2+2(a_3-a_5)u+a_6-a_7}.v$$

$$=\frac{8a_0u^3+4a_3u^2+2a_6u+a_8}{4(a_1-a_2)u^2+2(a_3-a_5)u+a_6-a_7}.$$

From each of these solutions it is obvious that the original equation has an integrating factor of the form $f(x+y)$.

II. Solution by E. B. ESCOTT, Ann Arbor, Mich.

Call the given equation $Mdx+Ndy=0\dots(2)$.
If F is an integrating factor, we must have

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=\frac{N\frac{\partial F}{\partial x}-M\frac{\partial F}{\partial y}}{F}\dots(3).$$

From symmetry of M and N, we may suppose F to be symmetrical in x and y. Suppose F is a function of $x+y$. Then

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}=\frac{\partial F}{\partial z},$$

and equation (3) becomes an ordinary differential equation instead of a partial differential equation. (3) becomes when the values of M and N are substituted,

$$(4a_1-3a_0-3a_2)(x^2-y^2)+(a_4-2a_5)(x-y)=$$
$$-[(a_1-a_2)(x+y)^2+(a_3-a_5)(x+y)+a_6-a_7](x-y)\frac{\frac{dF}{dz}}{F},$$

or, dividing by $x-y$ and putting $x+y=z$,

$$(4a_1-3a_0-3a_2)z+a_4-2a_5=-[(a_1-a_2)z^2+(a_3-a_5)z+a_6-a_7]\frac{dF}{dz}\cdot\frac{1}{F}.$$

The variables are separable, and we have

$$\frac{dF}{F}=-\frac{(4a_1-3a_0-a_2)z+a_4-2a_5}{(a_1-a_2)z^2+(a_3-a_5)z+a_6-a_7}dz=\left(\frac{a}{z-\alpha}+\frac{b}{z-\beta}\right)dz.$$

Then $F=(z-\alpha)^a(z-\beta)^b$, etc.

291. Proposed by V. M. SPUNAR, M. and E. E., Pittsburg, Pa.

Integrate $\frac{dy}{dx}=ay^2+bx^m$.